Critical Reasoning 14 - The Logic of Relations

Gottfried Wilhelm von Leibniz, (1646 - 1716) More Often Remembered for his Independent Invention of Calculus, was also a Philosopher and Logician of Note. Seen here in a Portrait housed at the Regional Library of Hannover.

Most of what counts for Science or indeed Humanities cannot be expressed solely in terms of the predicate calculus we have learned so far. This is because these disciplines proceed by discovering relations among things. A relation then, is a connection that one (or more) things or phenomena have to others. In the case that they involve two objects, they can be expressed by, so called, **binary predicates** (with two subjects) that represent the relation that subsists between them. Thus:

John loves Sarah

Even though "Sarah" is the grammatical object of the sentence above, both "John" and "Sarah" are regarded as the subjects of the predicate "… loves…". Of course some relations like love can be **reflexive** in that they relate to themselves, such as:

John loves himself

Quite naturally speaking, there are several *triadic* or even *tetradic* predicates that relate three or for subjects respectively. *E.g.*

George is between Cape Town and Port Elizabeth. and

Jack traded his cow to the peddler for some magic beans.

Examples of polyadic predicates involving more than four relations do exist but tend to result from longer lists of subject rather than naturally spoken relations, although sentences such as the following are the exception:

Jean and Murray played Diane and Ben at bridge over a bowl of popcorn.

As Copi (p. 117) observes, relations enter into arguments in various ways, such as:

Al is older than Bill.
Bill is older than Charlie.
Therefore, Al is older than Charlie.

This is simply a relational argument in which other relational predicates could have featured, such as "…is taller than..," "…is wiser than..," "… is heavier than…" *etc.* Consider the following, slightly more complex argument involving quantification:

Helen likes David.
Whoever likes David likes Tom.

Helen likes only good-looking men.
Therefore, Tom is a good-looking man.

The following more complex relational argument employed by Copi (*l.c.*) meanwhile involves multiple quantification.

All horses are animals.
Therefore, the head of a horse is the head of an animal.

Fortunately, no novel methods of proof, other than those already encountered in Critical Reasoning 11, are required to prove the validity of such arguments. However we must pay particular attention to the way we symbolise relational propositions.

Symbolising Relations

So far we have used only one-place predicates of the form Fx to represent "x is fast". Now we can expand this notation to represent predicates of the form:

x is taller than y as: Txy and

x is between y and z as: $Bxyz$.

Although other publications follow the convention of symbolising binary or dyadic as "xTy" and so on, we shall stick with Copi's notation, if only for consistency.

Just as we regarded different substitution instances of the same predicate "Fx", as representing different propositions, so we may regard different substitution instances of the same polyadic predicate *e.g. "$Bxyz$",* as representing different propositions. Note that the order of substitution is very important here. If the predicate "x taught y" is substituted in terms of "x" for "Socrates" and "y" for "Plato", the proposition:

Tsp is true,

whereas the proposition:

Tps is false.

According to the meaning of a proposition we may be required to use either existential or universal quantification, either singly or in combination, when symbolising it. Either way we should pay especial heed to which variables are free and which are bound (see Critical Reasoning 11 p.6) as well as the scope of our quantifiers. Do they extend across the whole expression or only part of it? The following examples were compiled from lecture notes as well as Copi (p. 119 ff.) Where the meaning of an expression may not be immediately apparent, a paraphrasal is suggested.

1. Something is taller than everything.

   $(\exists x)(\forall y)Txy$

   Paraphrasal: There exists an x, such that for all y, x is taller than y.

2. George is between Cape Town and something or other.

$(\exists x)gcx$

3. There is something between Cape Town and Port Elizabeth.

   $(\exists x)Bxcp$

4. There is something, such that that George is between that thing and Port Elizabeth.

   $(\exists x)gxp$

5. Everything attracts everything.

   $(\forall x)(\forall y)Axy$

   Paraphrasal: For any x and for any y, x attracts y.

6. Everything is attracted by everything.

   $(\forall y)(\forall x)Axy$

   Paraphrasal: For any y and for any x, x attracts y. (The passive voice of 5, above)

7. Something attracts something.

   $(\exists x)(\exists y)Axy$

8. Something attracts nothing.

   $(\exists x)(\forall y){\sim}Axy$

9. Alice is attracted by something.

   $(\exists x)Axa$

10. Nothing attracts Bert.

    $(\forall x){\sim}Axb$

11. Something is attracted by itself.

    $(\exists x)Axx$

12. Nothing attracts itself.

    $(\forall x){\sim}Axx$

Exercise: Now that we have seen how some common relations are symbolised, you may wish to try your hand at symbolising some of the more challenging sentences provided by Copi, section 5.1 part II (p. 128). The first ten are reproduced below.

1. Dead men tell no tales.
2. A lawyer who pleads his own case has a fool for a client.
3. A dead lion is more dangerous than a live dog.
4. Uneasy lies the head that wears the crown.

5. If a boy tells only lies, none of them will be believed.
6. Anyone who consults a psychiatrist ought to have his head examined.
7. No one ever learns anything unless he teaches it to himself.
8. Delilah wore a ring on every finger and had a finger in every pie.
9. Any man who hates children and dogs cannot be all bad.
10. Anyone who accomplishes anything will be envied by everyone.

Solutions: The following solutions, some of which are not the same as Copi's, were provided by Professor Peter Suber in the form of a hand-out. The full set of 45 translations is available here. They are reproduced below with additional paraphrasals. Note that the format adopted by Suber is known as "**prenex normal form**". Accordingly, "all the quantifiers in a formula are stacked at the left end, none is negated, and the scope of each extends to the end of the whole formula." (*l.c.*) This is not the only preferred form. Indeed many logicians do not follow a consistent format, preferring to preserve the form of natural language wherever possible.

1. $(\forall x)(\forall y)[(Dx \bullet Mx \bullet Ty) \supset \sim Txy]$

   Paraphrasal: For all x and for all y, such that if x is dead and x is a man and y is a tale, then x does not tell y.

2. $(\forall x)(\forall y)[(Lx \bullet Pxx) \supset (Cyx \bullet Fy)]$

   Paraphrasal: For all x and for all y, such that if x is a lawyer and x pleads x's case (*i.e.* his own), then y has x as a client and y is a fool.

   Note that the meaning of the proposition is preserved in this formulation, however we only appreciate the wit when we realise that the lawyer and client are the same person. This can be made explicit as follows: $(\forall x)[(Lx \bullet Pxx) \supset (Cxx \bullet Fx)]$

3. $(\forall x)(\forall y)[(\sim Ax \bullet Lx \bullet Ay \bullet Dy) \supset Dxy]$

   Paraphrasal: For all x and for all y, such that if x is not alive and x is a lion and y is alive and y is a dog, then x is more dangerous than y.

4. $(\forall x)(\exists y)[(Hx \bullet Cy \bullet Wxy) \supset Ux]$

   Paraphrasal: For all x and there exists a y, such that if x is a head and y is a crown and x wears y, then x lies uneasy.

5. $(\forall x)(\forall y)(\forall z)(\forall u)\{[Bx \bullet (Txy \supset Ly)] \supset (Txy \supset \sim Buz)\}$

   Paraphrasal: For all x, y, z and u, if x is a Boy and if x tells y such that y is a lie, then if x tells y then u will not believe z.

6. $(\forall x)(\exists y)[(Px \bullet Sy \bullet Cxy) \supset Ox]$

   Paraphrasal: For all x and there exists some y, then if x is a person and y is a psychiatrist and x consults y then x ought to have his head examined.

7. $(\forall x)(\exists y)[(Px \supset \sim Lxy) \supset \sim Txyx]$

Paraphrasal: For all x and there exists some y, such that if x is a person and x did not learn y, then x did not teach y to x (himself).

8. $(\forall x)(\exists y)[(Fxd \bullet Ry) \supset Oyx] \bullet (\exists x)(\forall y)[(Fxd \bullet Py) \supset Ixy]$

   Paraphrasal: For all x and there exists some y, such that x is a finger of Delilah's and y is a ring then y is on x, and there exists some x and for all y, such that x is a finger of Delilah's and y is a pie, then x is in y.

   Note that this is a conjunction between two quantified statements. Unlike the prenex normal form, none of the quantifiers' scope extends across the whole formula.

9. $(\forall x)(\forall y)(\forall z)[(Mx \bullet Cy \bullet Dz \bullet Hxy \bullet Hxz) \supset \sim Bx]$

   Note: Copi treats "x is all bad" as a predicate so there is no need to quantify badness.

   Paraphrasal: For all x, y and z such that if x is a man, y is a child and z is a dog and x hates y and x hates z, then x is not all bad.

10. $(\forall x)(\exists y)(\forall z)[(Px \bullet Pz \bullet Axy) \supset Ezx]$

    Paraphrasal: For all x and there exists some y and for all z such that if x is a person and z is a person and x accomplishes y, then z envies x.

    Note that Copi's sentence allows that any and every thing that is accomplished will result in the accomplisher being envied by all persons, including themselves.

Arguments Involving Relations

As mentioned, no new techniques are required in dealing with arguments involving relations. The four quantification inferences learned in Critical Reasoning 11 now take on the following forms:

Universal Instantiation (EG)

$$\frac{(\forall x)Fx}{\therefore Fx}$$

Universal Generalisation (UG)

$$\frac{Fx}{\therefore (\forall x)Fx}$$

Existential Instantiation (EG)

$$\begin{array}{l} (\exists x)Fx \\ \rightarrow Fx \\ \quad \vdots \\ \quad p \\ \hline \therefore p \end{array}$$

Existential Generalisation (EG)

$$\frac{Fy}{\therefore (\exists y)Fy}$$

Unlike the previous quantification rules, here we do not require that μ and ν be different variables. Instead we can simply instantiate with respect to the same variable in a premise. Of course the other restrictions apply, such as the correct nesting of assumptions. Similarly, where we have two premises "$(\exists x)Fx$" and "$(\exists x)\sim Fx$" and we begin our instantiation of one by simply dropping the quantifier, then we must instantiate the other in terms another letter if it is within the scope of the first instantiation. (Copi, p. 131)

Consider, by way of example, the following proofs of validity from Copi (Ex II p. 133-4) again provided by Professor Suber (available here.)

2. All circles are figures. Therefore all who draw circles draw figures.

1.	$(\forall x)[Cx \supset Fx]$ /∴ $(\forall x)(\forall y)[(Cx \bullet Dyx) \supset (Fx \bullet Dyx)]$	
→ 2.	$Cx \bullet Dyx$	
3.	Cx	2 Simp.
4.	Dyx	2 Simp.
5.	$Cx \supset Fx$	1 UI
6.	Fx	5,3 M.P.
7.	$Fx \bullet Dyx$	6,4 Conj.
8.	$(Cx \bullet Dyx) \supset (Fx \bullet Dyx)$	2-7 CP
9.	$(\forall y)[(Cx \bullet Dyx) \supset (Fx \bullet Dyx)]$	8 UG
10.	$(\forall x)(\forall y)[(Cx \bullet Dyx) \supset (Fx \bullet Dyx)]$	9 UG

5. Whoever belongs to the Country Club is wealthier than any member of the Elks Lodge. Not everyone who belongs to the Country Club is wealthier than anyone who does not belong. Therefore, not everyone belongs to either to the Country Club or the Elks Lodge.

   Paraphrasal: For all x and for all y such that if x belongs to the Country Club and y belongs to the Elks Lodge then x is wealthier than y. There exists an x and there exists a y such that x belongs to the Country Club and y does not belong to the Country Club such that x is not wealthier than y. (*I.e.* At least one member of the Country Club is not wealthier than at least one non-member.) Therefore, there exists an x that is neither a member of the Country Club nor a member of the Elks Lodge.

1.	$(\forall x)(\forall y)[(Cx \bullet Ey) \supset Wxy]$	
2.	$(\exists x)(\exists y)(Cx \bullet \sim Cy \bullet \sim Wxy)$	/∴ $(\exists x)(\sim Cx \bullet \sim Ex)$
→ 3.	$(\exists y)(Cx \bullet \sim Cy \bullet \sim Wxy)$	
→ 4.	$Cx \bullet \sim Cy \bullet \sim Wxy$	
5.	$(\forall y)[(Cx \bullet Ey) \supset Wxy]$	1 UI
6.	$(Cx \bullet Ey) \supset Wxy$	5 UI
7.	Cx	4 Simp.
8.	$\sim Cy$	4 Simp.
9.	$\sim Wxy$	4 Simp.
10.	$\sim(Cx \bullet Ey)$	6,9 M.T.
11.	$\sim Cx \vee \sim Ey$	10 De M.
12.	$\sim Ey$	11,7 D.S.
13.	$\sim Cy \bullet \sim Ey$	8,12 Conj.
14.	$(\exists x)(\sim Cx \bullet \sim Ex)$	13 EG
15.	$(\exists x)(\sim Cx \bullet \sim Ex)$	3,4-14 EI
16.	$(\exists x)(\sim Cx \bullet \sim Ex)$	2,3-15 EI

7. Everything on my desk is a masterpiece. Anyone who writes a masterpiece is a genius. Someone very obscure wrote some of the novels on my desk. Therefore, some very obscure person is a genius.

1.	$(\forall x)(Dx \supset Mx)$	
2.	$(\forall x)(\forall y)[(Px \bullet My \bullet Wxy) \supset Gx]$	
3.	$(\exists x)(\exists y)(Px \bullet Ox \bullet Ny \bullet Dy \bullet Wxy)$	$/\therefore (\exists x)(Px \bullet Ox \bullet Gx)$
4.	$(\exists y)(Px \bullet Ox \bullet Ny \bullet Dy \bullet Wxy)$	
5.	$Px \bullet Ox \bullet Ny \bullet Dy \bullet Wxy$	
6.	$Dy \supset My$	1 UI
7.	Dy	5 Simp.
8.	My	6,7 M.P.
9.	Px	5 Simp.
10.	Wxy	5 Simp.
11.	$Px \bullet My \bullet Wxy$	9,8,10 Conj.
12.	$(\forall y)[(Px \bullet My \bullet Wxy) \supset Gx]$	2 UI
13.	$(Px \bullet My \bullet Wxy) \supset Gx$	12 UI
14.	Gx	13,11 M.P.
15.	Ox	5 Simp.
16.	$Px \bullet Ox \bullet Gx$	9,15,14 Conj.
17.	$(\exists x)(Px \bullet Ox \bullet Gx)$	16 EG
18.	$(\exists x)(Px \bullet Ox \bullet Gx)$	4,5-17 EI
19.	$(\exists x)(Px \bullet Ox \bullet Gx)$	3,4-18 EI

Jon Ross has provided more proofs to this and further exercises in Copi's text. The interested reader can find the relevant introduction and links here or direct links to the remaining chapters as follows : chapter 5, chapter 7, and Chapter 8.

Some Attributes of Binary or Dyadic Relations

Relations themselves have several interesting attributes, some of which may enter into proofs. For now we shall confine ourselves to binary or dyadic relations, however there is no upper limit on the number of attributes of polyadic relations.

1. Symmetry: A relation is symmetric if the following is true: $(\forall x)(\forall y)(Rxy \supset Ryx)$

Paraphrasal: If x has a relation R to y, then y has relation R to x.

E.g. "x is next to y", or "x is married to y", or "a is the twin of b" *etc.*

On the other hand, a relation is **asymmetrical** if the following is true $(\forall x)(\forall y)(Rxy \supset \sim Ryx)$

Paraphrasal: If x has a relation R to y, then y does not have a relation R to x.

E.g. "x is older than y", or "x is North of y" *etc.*

Meanwhile, a relation is **non-symmetrical** if it is neither symmetrical nor asymmetrical.

E.g. "x is a sister of y", or "x loves y" *etc.*

2. Transitivity: A relation is transitive if the following is true: $(\forall x)(\forall y)(\forall z)[(Rxy \bullet Ryz) \supset Rxz]$

Paraphrasal: If x has a relation R to y and y has the same relation R to z, then x has the same relation R to z.

E.g. "x is North of y", or "x is larger than y", or "S_1 predates S_2" *etc.*

On the other hand, a relation is **intransitive** if: $(\forall x)(\forall y)(\forall z)[(Rxy \bullet Ryz) \supset {\sim}Rxz]$

Paraphrasal: If x has a relation R to y and y has the same relation R to z, then x cannot have the same relation R to z.

E.g. "x is the father of y", or "x weighs exactly twice as much as y" *etc.*

Meanwhile, a relation is **non-transitive** if it is neither transitive nor intransitive.

E.g. "x loves y", or "x has a different mass to y"

3. Reflexivity: A relation Rxy is **totally reflexive** if the following is true: Rxx

E.g. "x is identical to itself"

On the other hand, a relation Rxy is **reflexive** if: $(\forall x)[(\exists y)(Rxy \vee Ryx) \supset Rxx]$

Paraphrasal: x has a relation R to itself if there is something y such that either Rxy or Ryx.

E.g. "x is the same age as y", or "x is a contemporary of y", or "x has the same eye colour as y" *etc.*

Meanwhile, a relation Rxy is **irreflexive** if the following is true: $(\forall x){\sim}Rxx$

Paraphrasal: x cannot have a relation R to itself.

E.g. "x is North of y", or "x is married to y", or "$x > y$ for Real numbers"

Finally, a relation is **non-reflexive** if it is not totally reflexive, reflexive or irreflexive.

E.g. "x loves y", or "x hates y" *etc.*

Note: Most binary or dyadic relations have more than one property or attribute. *E.g.* the relation: "… weighs more that…" is at the same time asymmetrical, transitive and irreflexive, while that of "… weighs the same as…" is symmetrical, transitive and reflexive. (Copi, p. 136) See if you can identify the attributes in the following relations.

1. x is uncle of y.
2. $x = y$
3. x is as big as y.
4. x is a sister of y.

Solutions:

1. "x is uncle of y" is at the same time:

   (a) asymmetrical: $(\forall x)(\forall y)(Uxy \supset {\sim}Uyx)$
   (b) intransitive: $(\forall x)(\forall y)(\forall z)[(Uxy \bullet Uyz) \supset {\sim}Uxz]$ and
   (c) irreflexive: $(\forall x){\sim}Uxx$

2. "$x = y$" is at the same time:

(a) symmetrical: $(\forall x)(\forall y)[(x = y) \supset (y = x)]$
(b) transitive: $(\forall x)(\forall y)(\forall z)\{[(x = y) \bullet (y = z)] \supset (x = z)\}$ and
(c) totally reflexive: $(\forall x)(x = x)$

3. "x is as big as y" is at the same time:

(a) symmetrical: $(\forall x)(\forall y)(Bxy \supset Byx)$
(b) transitive: $(\forall x)(\forall y)(\forall z)[(Bxy \bullet Byz) \supset Bxz]$ and
(c) reflexive: $(\forall x)[(\exists y)(Bxy \text{ v } Byx) \supset Bxx]$

4. "x is a sister of y"

(a) non-symmetrical: neither symmetrical nor asymmetrical - no symmetrical relation
(b) transitive: $(\forall x)(\forall y)(\forall z)[(Sxy \bullet Syz) \supset Sxz]$
(c) irreflexive: $(\forall x){\sim}Sxx$

Copi (p.136) points out furthermore, that some attributes entail the presence of others, such as that all asymmetrical relations must also be irreflexive. This he derives as follows:

1. $(\forall x)(\forall y)(Rxy \supset {\sim}Ryx)$	Definition of asymmetry
2. $(\forall y)(Rxy \supset {\sim}Ryx)$	1 UI
3. $Rxy \supset {\sim}Ryx$	2 UI
4. ${\sim}Rxx \text{ v } {\sim}Rxx$	3 Impl.
5. ${\sim}Rxx$	4 Taut.
6. $(\forall x){\sim}Rxx$	5 UG

This last line, as can be seen, is the definition of irreflexivity and so the proof is complete.

Note: When using models to prove invalidity, such as those introduced in Critical Reasoning 11, we must be cautious when assessing arguments that involve relational attributes. We cannot, for example, consistently begin with a model $\boxed{a}$ containing only one object when a relation of irreflexivity, $(\forall x){\sim}Rxx$ obtains. There must be at least two non-identical objects. Similarly with the relation of transitivity, $(\forall x)(\forall y)(\forall z)[(Rxy \bullet Ryz) \supset Rxz]$ a potentially consistent model must contain at least three non-identical objects.

One problem with relational attributes is precisely that they are assumed and therefore unstated. In Critical Reasoning 01 we called such arguments enthymemes. Where an enthymeme has a premise that is supressed or "understood", therefore we shall have to take it (them) into account when considering validity. (Copi p. 136-137)

Enthymemes

Consider the argument on page 1 above:

Al is older than Bill.
Bill is older than Charlie.
Therefore, Al is older than Charlie.

Technically it is invalid as it stands, but we account it valid just the same because the relationship of "… being older than…" has the obvious, but unstated, attribute of transitivity. Sometimes however the missing or auxiliary premise may be about factual linguistic assumptions. Consider:

All dogs are tame.
Therefore, Fido is tame.

Clearly we are assuming that Fido is a dog (Df) and so we have:

1. $(\forall x)(Dx \supset Tx)$ $/\therefore Tf$
2. Df (auxiliary premise)
3. $Df \supset Tf$ 1 UI
4. Tf 3,2 M.P.

Even within the same relatively simple argument we might make multiple assumptions about relations and matters of fact, as in the following example by Copi (p. 138-139).

Any horse can outrun any dog.
Some greyhounds can outrun any rabbit.
Therefore, any horse can outrun any rabbit.

Here we are making the assumption that the relation "… can outrun…" is a transitive one and the fact that all greyhounds are dogs. This makes the argument only a little longer to state but more lengthy to prove. Here is Copi's proof of the same:

1. $(\forall x)[Hx \supset (\forall y)(Dy \supset Oxy)]$
2. $(\exists y)[Gy \bullet (\forall z)(Rz \supset Oyz)]$ $/\therefore (\forall x)[Hx \supset (\forall z)(Rz \supset Oxz)]$
3. $(\forall x)(\forall y)(\forall z)[(Oxy \bullet Oyz) \supset Oxz]$ (auxiliary premise of transitivity)
4. $(\forall y)(Gy \supset Dy)$ (auxiliary premise that all greyhounds are dogs)
5. Hx
6. Rz
7. $Gy \bullet (\forall z)(Rz \supset Oyz)$
8. Gy 7 Simp.
9. $Gy \supset Dy$ 4 UI
10. Dy 9,8 M.P.
11. $Hx \supset (\forall y)(Dy \supset Oxy)$ 1 UI
12. $(\forall y)(Dy \supset Oxy)$ 11,5 M.P.
13. $Dy \supset Oxy$ 12 UI
14. Oxy 13,10 M.P.
15 $(\forall z)(Rz \supset Oyz)$ 7 Simp.
16. $Rz \supset Oyz$ 15 UI
17. Oyz 16,6 M.P.
18. $Oxy \bullet Oyz$ 14,17 Conj.
19. $(\forall y)(\forall z)[(Oxy \bullet Oyz) \supset Oxz]$ 3 UI
20. $(\forall z)[(Oxy \bullet Oyz) \supset Oxz]$ 19 UI
21. $(Oxy \bullet Oyz) \supset Oxz$ 20 UI
22. Oxz 21,18 M.P.
23. Oxz 2,7-22 EI
24. $Rz \supset Oxz$ 6-23 C.P.
25. $(\forall z)(Rz \supset Oxz)$ 24 UG
26. $Hx \supset (\forall z)(Rz \supset Oxz)$ 5-25 C.P.

27. $(\forall x)[Hx \supset (\forall z)(Rz \supset Oxz)]$ 26 UG

Exercise: If you have a copy of Copi's textbook at hand you may wish to try and prove a selection of some of the enthymemes from the exercise at the end section 5.3 on pages 139-140. Otherwise, here are the first two. The solutions below were provided by John Ross and are available here.

1. A Cadillac is more expensive than any low-priced car. Therefore, no Cadillac is a low-priced car. (Cx: x is a Cadillac. Lx: x is a low-priced car. Mxy: x is more expensive than y.)

1.	$(\forall x)[Cx \supset (\forall y)(Ly \supset Mxy)]$	$/\therefore (\forall x)(Cx \supset \sim Lx)$
2.	$(\forall x)\sim Mxx$	(auxiliary premise of irreflexivity)
→ 3.	Cx	
4.	$Cx \supset (\forall y)(Ly \supset Mxy)$	1 UI
5.	$(\forall y)(Ly \supset Mxy)$	4,3 M.P.
6.	$Lx \supset Mxx$	5 UI
7.	$\sim Mxx$	2 UI
8.	$\sim Lx$	6,7 M.T.
9.	$Cx \supset \sim Lx$	3-8 C.P
10.	$(\forall x)(Cx \supset \sim Lx)$	9 UG

2. Alice is Betty's mother. Betty is Charlene's mother. Therefore, if Charlene loves only her mother, then she does not love Alice. (a: Alice. b: Betty. c: Charlene. Mxy: x is mother of y. Lxy: x loves y.

1.	Mab	
2.	Mbc	$/\therefore (\forall x)(Lcx \supset Mxc) \supset \sim Lca$
3.	$(\forall x)(\forall y)(\forall z)[(Mxy \bullet Myz) \supset \sim Mxz]$	(auxiliary premise of intransitivity)
→ 4.	$(\forall x)(Lcx \supset Mxc)$	
5.	$(\forall y)(\forall z)[(Mxy \bullet Myz) \supset \sim Mxz]$	3 UI
6.	$(\forall z)[(Mxy \bullet Myz) \supset \sim Mxz]$	5 UI
7.	$(Mab \bullet Mbc) \supset \sim Mac$	6 UI
8.	$Lcx \supset Mxc$	4 UI
9.	$\sim Mac \supset \sim Lac$	8 Trans.
10.	$(Mab \bullet Mbc) \supset \sim Lac$	7,9 H.S.
11.	$Mab \bullet Mbc$	1,2 Conj.
12.	$\sim Lac$	10, 11 M.P.
12.	$(\forall x)(Lcx \supset Mxc) \supset \sim Lca$	4-12 C.P.

Identity

The special relation of identity, symbolised by the "=" sign merits distinctive treatment both because of its importance and ubiquity. According to **Leibniz's Law**, two entities are identical if and only if they have all their properties in common. Also known as the **Law of Identity** or **Identity of Indiscernibles**, it is symbolised as:

$$(\forall x)(\forall y)[(x = y) \equiv (\forall F)(Fx \equiv Fy)]$$

This formulation of identity is much stronger that of our informal use of the term when, for example referring to "identical" twins. Even if two twins, say Bobby and Joe, are truly indiscernible even by their parents, one would have the property of having one name, the other another. One may have the property of standing on the left, the other on the right. One may have the property of having been born before the other, and so on. Therefore, they are not logically identical.

We should also pause here to point out that quantifying over properties as we have done here is regarded as illegitimate by a significant proportion of philosopher-logicians for reasons that will take us too far from the present topic. Suffice it to say that you should regard Leibniz's Law and what follows as given, until such time as you have developed your own reasoned position on the matter.

Rules of Identity (Id.)

$$\frac{\begin{array}{l}\Phi\mu \\ \nu = \mu\end{array}}{\therefore \Phi\nu} \qquad \frac{\begin{array}{l}\Phi\mu \\ \sim(\Phi\nu)\end{array}}{\therefore \sim(\nu = \mu)} \qquad \frac{\nu = \mu}{\therefore \mu = \nu} \qquad \frac{p}{\therefore \mu = \mu}$$

Consider the following instances of use of the rules of identity, left to right:

1. Dollar Brand is a musician. Dollar Brand is Abdullah Ibrahim. Therefore, Abdullah Ibrahim is a musician.

   1. Md
   2. $d = a$ $\quad /\therefore Ma$
   3. Ma $\quad$ 1,2 Id.

2. Dollar Brand is talented. Phil is not talented. Therefore, Dollar Brand is not Phil.

   1. Td
   2. $\sim Tp$ $\quad /\therefore d \neq p$
   3. $d \neq p$ $\quad$ 1,2 Id.

3. Dollar Brand is Abdullah Ibrahim. Therefore, Abdullah Ibrahim is Dollar Brand.

   $d = a$ $\quad /\therefore a = d$ (Identity is symmetrical.)

4. If Dollar Brand is identical to himself, then Phil is South African. Therefore, Phil is South African.

   1. $(d = d) \supset Sp$ $/\therefore Sp$
   2. $d = d$ $\quad$ 1. Id. (Identity is totally reflexive.)
   3. Sp $\quad$ 1,2 M.P.

Note that in the above examples identity is not used on only part of line. Also observe the shorthand "$\nu \neq \mu$" for "$\sim(\nu = \mu)$".

Examples of sentences involving the identity predicate: Px: x is a person. j= Jane; Txy: x is taller than y.

1. Jane is taller than anyone else.

$(\forall x)[(Px \bullet x \neq j) \supset Tjx]$

2. There is someone who is taller than any other person.

   $(\exists x)\{Px \bullet (\forall y)[(Py \bullet y \neq x) \supset Txy]\}$

3. Jane is the only intelligent person.

   $(\forall x)[(Px \bullet Ix) \supset x = j]$ or $(\forall x)[Px \bullet x \neq j) \supset \sim Ix]$

4. There is only one intelligent person.

   $(\exists x)\{(Px \bullet Ix) \bullet (\forall y)[(Py \bullet Iy) \supset y = x]\}$

   Note that the left hand side of the "•" with the largest scope ensures that there is at least one such person, while the expression to the right ensures that there is at most one.

5. There are exactly two intelligent persons.

   $(\exists x)(\exists y)\{(Px \bullet Ix \bullet Py \bullet Iy \bullet x \neq y) \bullet (\forall z)[(Pz \bullet Iz) \supset (z = x \vee z = y)]\}$

   Note again, how in similar fashion, the left hand side of the "•" with the largest scope insures that there are at least two such persons, while the expression to the right ensures that there are at most two. There is no upper limit on how logic can "count" in this way, except to observe that the expressions very quickly become increasingly cumbersome.

6. Al is on the team and can outrun anyone else on it. a = Al; Tx: x is on the team. Oxy: x can outrun y. If, in this example by Copi, (p. 143) we were to write: $Ta \bullet (\forall x)(Tx \supset Oax)$, it would entail that Al can outrun himself, whereas we know that *being able to outrun* is an irreflexive relation. What we have to capture is the meaning of the word "else", hence:

   $Ta \bullet (\forall x)[(Tx \bullet x \neq a) \supset Oax]$

Arguments Involving Identity

Consider the following examples of proofs involving identity before trying your own hand at nos. 5, 6, 9 and 10 from the exercise at the end of section 5.4 of Copi (p. 150):

1. Bill is Smith. Smith is taller than Jones. Therefore, Bill is taller than someone or other. (Note that the "is" in the first premise is one of identity not predication.)

   1. $b = s$
   2. Tsj $\quad /\therefore (\exists x)Tbx$
   3. Tbj $\quad$ 1,2 Id.
   4. $(\exists x)Tbx$ $\quad$ 3 E.G.

2. 1. $(\forall x)(\forall y)(x = y \supset Rxy)$
   2. $\sim Rcb$
   3. $a = x$ $\quad /\therefore a \neq b$
   4. $(\forall y)(a = y \supset Ray)$ $\quad$ 1 UI
   5. $a = b \supset Rab$ $\quad$ 4 UI

6. $\sim Rab$ 2,3 Id.
7. $a \neq b$ 5,6 M.T.

3.
1. Fa
2. $(\exists x)(Gx \bullet a = x)$ /∴ $(\exists x)(Fx \bullet Gx)$
→ 3. $Gx \bullet a = x$
4. $a = x$ 3 Simp.
5. Fx 1,4 Id.
6. Gx 3 Simp.
7. $Fx \bullet Gx$ 5,6 Conj.
8. $(\exists x)(Fx \bullet Gx)$ 7 E.G.
9. $(\exists x)(Fx \bullet Gx)$ 2,3-8 EI

4. Only Jane is intelligent. Someone who is intelligent is tall. Therefore Jane is tall.

1. $(\forall x)(Ix \supset x = j)$
2. $(\exists x)(Ix \bullet Tx)$ /∴ Tj
→ 3. $Ix \bullet Tx$
4. $Ix \supset x = j$ 1 UI
5. Ix 3 Simp.
6. $x = j$ 4,5 M.P.
7. Tx 3 Simp.
8. Tj 7,6 Id.
9. Tj

Selected Solutions to Copi (p. 150)

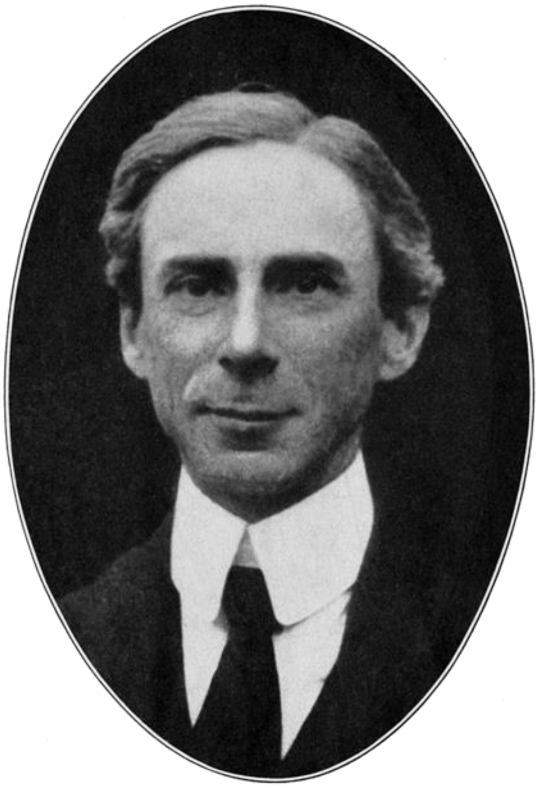
5. All entrants will win. There will be, at most, one winner. There is at least one entrant. Therefore, there is exactly one entrant. (Ex: x is an entrant. Wx: x will win.)

1. $(\forall x)(Ex \supset Wx)$
2. $(\forall x)(\forall y)[(Wx \bullet Wy) \supset y = x]$ (note the standard translation for "at most one")
3. $(\exists x)Ex$ /∴ $(\exists x)[Ex \bullet (\forall y)(Ey \supset y = x)]$ (and for "there is exactly one")
→ 4. Ex
5. $Ex \supset Wx$ 1 UI
6. Wx 5,4 M.P.
7. $(\forall y)[(Wx \bullet Wy) \supset y = x]$ 2 UI
8. $(Wx \bullet Wy) \supset y = x$ 7 UI
9. $Wx \supset (Wy \supset y = x)$ 7 Exp.
10. $Wy \supset y = x$ 9,6 M.P.
11. $Ey \supset Wy$ 1 UI (we have already used UI on line 5, now we must use y)
12. $Ey \supset y = x$ 11,10 H.S.
13. $(\forall y)(Ey \supset y = x)$ 12 UG
14. $Ex \bullet (\forall y)(Ey \supset y = x)$ 4,13 Conj.
15. $(\exists x)[Ex \bullet (\forall y)(Ey \supset y = x)]$ 14 EG
16. $(\exists x)[Ex \bullet (\forall y)(Ey \supset y = x)]$ 3,4-15 EI

9. There is exactly one penny in my right hand. There is exactly one penny in my left hand. Nothing is in both my hands. Therefore there are exactly two pennies in my hands. (Px: x is a penny. Rx: x is in my right hand. Lx: x is in my left hand.)

1. $(\exists x)\{Px \bullet Rx \bullet (\forall y)[(Py \bullet Ry) \supset y = x]\}$	
2. $(\exists x)\{Px \bullet Lx \bullet (\forall y)[(Py \bullet Ly) \supset y = x]\}$	
3. $(\forall x)(\sim Rx \vee \sim Lx)$	
$/\therefore (\exists x)(\exists y)[Px \bullet Py \bullet (Lx \vee Rx) \bullet (Ry \vee Ly) \bullet x \neq y \bullet (\forall z)\{[Pz \bullet (Lz \vee Rz)] \supset (z = x \vee z = y)\}]$	
4. $Pu \bullet Ru \bullet (\forall y)[(Py \bullet Ry) \supset y = u]$	
5. $Px \bullet Lx \bullet (\forall y)[(Py \bullet Ly) \supset y = x]$	
6. Pu	4 Simp.
7. Px	5 Simp.
8. Ru	4 Simp.
9. Lx	5 Simp.
10. $Ru \vee Lu$	8 Add.
11. $Lx \vee Rx$	9 Add.
12. $\sim Ru \vee \sim Lu$	3 UI
13. $Ru \supset \sim Lu$	12 Impl.
14. $\sim Lu$	13, 8 M.P.
15. $x \neq u$	9,14 Id.
16. $Px \bullet Pu$	7,6 Conj.
17. $Px \bullet Pu \bullet (Lx \vee Rx)$	16,11 Conj.
18. $Px \bullet Pu \bullet (Lx \vee Rx) \bullet (Ru \vee Lu)$	17,10 Conj.
19. $Px \bullet Pu \bullet (Lx \vee Rx) \bullet (Ru \vee Lu) \bullet x \neq u$	18,15 Conj.
20. $Pz \bullet (Lz \vee Rz)$	
21. $(\forall y)[(Py \bullet Ry) \supset y = u]$	4 Simp.
22. $(\forall y)[(Py \bullet Ly) \supset y = x]$	5 Simp.
23. $(Pz \bullet Rz) \supset z = u$	21 UI
24. $(Pz \bullet Lz) \supset z = x$	22 UI
25. $(Pz \bullet Lz) \vee (Pz \bullet Rz)$	20 Dist.
26. $[(Pz \bullet Lz) \supset z = x] \bullet [(Pz \bullet Rz) \supset z = u]$	24,23 Conj.
27. $z = x \vee z = u$	26,25 C.D.
28. $[Pz \bullet (Lz \vee Rz)] \supset (z = x \vee z = u)$	20-27 C.P.
29. $(\forall z)\{[Pz \bullet (Lz \vee Rz)] \supset (z = x \vee z = u)]\}$	28 UG
30. $Px \bullet Pu \bullet (Lx \vee Rx) \bullet (Ru \vee Lu) \bullet x \neq u \bullet (\forall z)\{[Pz \bullet (Lz \vee Rz)] \supset (z = x \vee z = u)\}$	19,29 Conj.
31. $(\exists y)[Px \bullet Py \bullet (Lx \vee Rx) \bullet (Ry \vee Ly) \bullet x \neq y \bullet (\forall z)\{[Pz \bullet (Lz \vee Rz)] \supset (z = x \vee z = y)\}]$	30 EG
32. $(\exists x)(\exists y)[Px \bullet Py \bullet (Lx \vee Rx) \bullet (Ry \vee Ly) \bullet x \neq y \bullet (\forall z)\{[Pz \bullet (Lz \vee Rz)] \supset (z = x \vee z = y)\}]$	31 EG
33. $(\exists x)(\exists y)[Px \bullet Py \bullet (Lx \vee Rx) \bullet (Ry \vee Ly) \bullet x \neq y \bullet (\forall z)\{[Pz \bullet (Lz \vee Rz)] \supset (z = x \vee z = y)\}]$	2,5-32 EI
34. $(\exists x)(\exists y)[Px \bullet Py \bullet (Lx \vee Rx) \bullet (Ry \vee Ly) \bullet x \neq y \bullet (\forall z)\{[Pz \bullet (Lz \vee Rz)] \supset (z = x \vee z = y)\}]$	1,4-33 EI

Russell's Theory of Descriptions


Bertrand Russell in 1916

What is the meaning of the word "the"? Despite "the" being the most commonly used word in the English language, very few of us (who haven't done a course in Linguistics or the Philosophy of Language) are likely to know. True, we know how to use it and *pace* Wittgenstein, for whom meaning *is* use, it was not until Bertrand Russell's 1905, now classic paper "On Denoting" (available here) published in the journal "Mind" that most Philosophers were in agreement about the logical structure and semantics of expressions involving denoting phrases.

Consider the following sorts of expressions:

1. Phrases that fail to denote, such as "the current King of France".
2. Phrases which do denote one object, such as "the President of the United states" which refers unambiguously to one known individual; or "the tallest spy" which refers unambiguously to one unknown individual.
3. Phrases which denote ambiguously, such as "an Aardvark". (Wikipedia: Theory of descriptions)

Russell's theory of descriptions is regarded as a paradigm of Philosophy because it apparently solved three problems simultaneously, namely:

- The problem of empty names
- How words attach to things, *i.e.* reference, and
- Truths (or falsehoods) that fail to refer.

In *On* Denoting (1905) Russell considers the sentence: "The current King of Face is bald." Clearly the sentence has a meaning that we understand but it fails to refer to anybody so how it possible for it to be either true or false? Russell analyses the sentence into the following components:

1. There is an x such that x is the current King of France.
2. For every x and y such that if x and y are current Kings of France then x is y. (*I.e.* there is at most one current King of France.)
3. Anything that is a current King of France is bald.

Thus, any definite description of the form "the F is B" can be expressed as:

$$(\exists x)[Fx \bullet (\forall y)(Fy \supset x = y) \bullet Bx]$$

According to Russell, this immediately solves two problems. Firstly, the sentence: "The current King of Face is bald" is straightforwardly seen to be false because there are no entities of which the predicate "… is the current King of France" is true. In fact all instances of "x has the property F" are false for all x, without x having to refer to any mysterious non-entity. Secondly, this solves Gottlob Frege's problem of informative identities; the identity of the "morning star" and the "evening star" being just such a case. Both have different meanings but the same referent *i.e.* "The planet Venus". Instead of treating a sentence like, "The morning star rises in the morning" as having a subject-predicate form, Russell analyses it as "there is one unique thing such that it is the morning star and it rises in the morning" which is not the same semantically as "there is one unique thing such that it is the evening star and it rises in the evening" even though both expressions ultimately have the same referent. (Wikipedia: Theory of descriptions)

Although few linguists would say that Russell has "solved" the question of reference, his theory of descriptions has recast such questions in terms of descriptions with which we may become familiar, rather than entities with which we are acquainted. Thus, I may learn about the existence of a special point, the centre of gravity of the Solar System, by its definite descriptions. Similarly, I might know about the not existence of other entities by their definite descriptions.

As far as indefinite descriptions are concerned, we know that the sentence "Some dog is annoying" is true, not because we are acquainted with the subject-predicate "Ad", but because of the existentially quantified conjunction of two descriptions:

1. There is an x such that x is a dog, and
2. x is annoying.

In symbols: $(\exists x)(Dx \bullet Ax)$ (Wikipedia: Theory of descriptions)

Paraphrasal: There exists an x such that x is a dog and x is annoying.

Arguments Involving Definite Descriptions

Some of the exercises at the end of Copi's section 5.4 involve definite descriptions, introduced by the article "the". Try first to symbolise, and then prove nos. 1-4 (p. 149-150) Again, the solutions below were provided by John Ross, available here. *E.g.*

1. The architect who designed Tappan Hall designs only office buildings. Therefore, Tappan Hall is an office building. (Ax: x is an architect. t: Tappan Hall. Dxy: x designed y. Ox: x is an office building.)

   1. $(\exists x)\{Ax \bullet Dxt \bullet (\forall y)[(Ay \bullet Dyt) \supset y = x] \bullet (\forall z)(Dxz \supset Oz)\}$ $/\therefore Ot$
   2. $Ax \bullet Dxt \bullet (\forall y)[(Ay \bullet Dyt) \supset y = x] \bullet (\forall z)(Dxz \supset Oz)$
   3. $(\forall z)(Dxz \supset Oz)$ 2 Simp.
   4. $Dxt \supset Ot$ 3 UI
   5. Dxt 2 Simp.
   6. Ot 4,5 M.P.
   7. Ot 1,2-6 EI

3. The smallest state is in New England. All states in New England are primarily industrial. Therefore, the smallest state is primarily industrial. (Sx: x is a state. Nx: x is in New England. Ix: x is primarily industrial. Sxy: x is smaller than y.

   1. $(\exists x)\{Sx \bullet (\forall y)[(Sy \bullet y \neq x) \supset Sxy] \bullet Nx\}$

2. $(\forall x)[(Sx \bullet Nx) \supset Ix]$ $/\therefore (\exists x)\{Sx \bullet (\forall y)[(Sy \bullet y \neq x) \supset Sxy] \bullet Ix\}$

→ 3.	$Sx \bullet (\forall y)[(Sy \bullet y \neq x) \supset Sxy] \bullet Nx$	
4.	$Sx \bullet Nx$	3 Simp.
5.	$(Sx \bullet Nx) \supset Ix$	2 UI
6.	Ix	5,4 M.P.
7.	$Sx \bullet (\forall y)[(Sy \bullet y \neq x) \supset Sxy]$	3.Simp.
8.	$Sx \bullet (\forall y)[(Sy \bullet y \neq x) \supset Sxy] \bullet Ix$	7,6 Conj.
9.	$(\exists x)\{Sx \bullet (\forall y)[(Sy \bullet y \neq x) \supset Sxy] \bullet Ix\}$	8 EG
10.	$(\exists x)\{Sx \bullet (\forall y)[(Sy \bullet y \neq x) \supset Sxy] \bullet Ix\}$	1,3-9 EI

Predicate Variables and Properties of Properties

So far we have been employing a **first order calculus** in which the Roman letters "F", "G", "H" *etc.* (but not the Greek letters "ϕ" and "ψ") have been treated as constants. Therefore we have treated an expression like "$(\exists x)Fx$" as a proposition, capable of being true or false, since it contains no free variables. On a **second order calculus** a predicate letter like "F" is treated as a variable. Therefore, on a second calculus, the expression "$(\exists x)Fx$" is considered to be a **propositional function**, incapable of being true or false because it is incomplete, containing the free variable, "F". To become a proposition, the expression would have to be written:

$$(\exists F)(\exists x)Fx \quad \text{or} \quad (\forall F)(\exists x)Fx.$$

Some standard translations of second order propositions (Copi, 151-152):

1. Socrates has all properties.

   $(\forall F)Fs$

2. Plato has some attribute.

   $(\exists F)Fp$

3. Everything has every attribute.

   $(\forall x)(\forall F)Fx$

4. Everything has some property or other.

   $(\forall x)(\exists F)Fx$

5. Something has every property.

   $(\exists x)(\forall F)Fx$

6. Every property belongs to something or other.

   $(\forall F)(\exists x)Fx$

7. Some property belongs to everything.

   $(\exists F)(\forall x)Fx$

8. Something has some property or other.

   $(\exists x)(\exists F)Fx$

Although such expressions are meaningful to us, it is a highly contentious question as to whether they could refer to anything in the external world outside our own mental constructs. There would almost certainly be no properties in a universe devoid of beings to think about them. Like Hume on causality, (Classic Text 04) we should be ontologically sceptical. However for example, we speak almost naturally about classes of particles having certain properties in common, or even being defined by their properties. Given that we do, a second order calculus must admit the possibility of properties having properties. Indeed it has to admit of the possibility of sentences such as the following (with second order predicates in bold):

1. All useful attributes are desirable.

   $(\forall F)(\boldsymbol{U}F \supset \boldsymbol{D}F)$

2. Some desirable attributes are not useful.

   $(\exists F)(\boldsymbol{D}F \bullet \sim\boldsymbol{U}F)$

3. Nothing which possesses every rare attribute has any ordinary attribute.

   $(\forall x)[(\forall F)(\boldsymbol{R}F \supset Fx) \supset (\forall G)(\boldsymbol{O}G \supset \sim Gx)]$

4. Nothing has all attributes.

   $\sim(\exists x)(\forall F)Fx$ or $(\forall x)(\exists F)\sim Fx$

5. Some attribute belongs to nothing.

   $(\exists F)(\forall x)\sim Fx$

6. Napoleon had all the attributes of a great general.

   $(\forall F)[(\forall x)(Gx \supset Fx) \supset Fn]$

Second order expressions involving the identity predicate can also be quantified. As Copi (p. 152) shows beginning with a purely symbolic definition of the identity, *i.e.*

$$(x = y) = df\ (\forall F)(Fx \equiv Fy)$$

from which it follows that,

$$(\forall x)(\forall y)[(x = y) \equiv (\forall F)(Fx \equiv Fy)]$$

which will be recognised as Leibniz's law. Consider the following two examples.

7. Any two things have some common attribute.

   $(\forall x)(\forall y)[x \neq y \supset (\exists F)(Fx \bullet Fy)]$

8. Jones and Smith share all their good qualities but have no bad quantities in common.

$(\forall F)[\boldsymbol{G}F \supset (Fs \equiv Fj)] \bullet (\forall F)\{\boldsymbol{B}F \supset [(Fs \supset {\sim}Fj) \bullet (Fj \supset Fs)]\}$

Finally, we can use the method of conditional proof learned in Critical Reasoning 09 to prove the following tautologies involving attributes or properties:

1. If circles are ellipses then circles have all the properties of ellipses.

   $(\forall x)(Cx \supset Ex) \supset (\forall F)[(\forall y)(Ey \supset Fy) \supset (\forall z)(Cz \supset Fz)]$

→1. $(\forall x)(Cx \supset Ex)$	
→2. $(\forall y)(Ey \supset Fy)$	
3. $Cz \supset Ez$	1 UI
4. $Ez \supset Fz$	2 UI
5. $Cz \supset Fz$	3,4 H.S.
6. $(\forall z)(Cz \supset Fz)$	5 UG
7. $(\forall y)(Ey \supset Fy) \supset (\forall z)(Cz \supset Fz)$	2-6 C.P.
8. $(\forall F)[(\forall y)(Ey \supset Fy) \supset (\forall z)(Cz \supset Fz)]$	7 UG
9. $(\forall x)(Cx \supset Ex) \supset (\forall F)[(\forall y)(Ey \supset Fy) \supset (\forall z)(Cz \supset Fz)]$	1-8 C.P.

2. All asymmetrical binary relations are irreflexive.

   Recall that for binary or dyadic relationships the relationship is asymmetrical if

   $(\forall x)(\forall y)(Rxy \supset {\sim}Ryx)$

   is true and irreflexive if

   $(\forall x){\sim}Rxx$

   is true. If we connect these as a conditional statement and then universally quantify over both we shall arrive at the desired translation:

   $(\forall R)[(\forall x)(\forall y)(Rxy \supset {\sim}Ryx) \supset (\forall x){\sim}Rxx]$

→1. $(\forall x)(\forall y)(Rxy \supset {\sim}Ryx)$	
2. $(\forall y)(Rxy \supset {\sim}Ryx)$	1 UI
3. $Rxx \supset {\sim}Rxx$	2UI
4. Rxx v ${\sim}Rxx$	3 Impl.
5. Rxx	4 Taut.
6. $(\forall x){\sim}Rxx$	5 UG (legitimate use of UG because x is bound in 1)
7. $(\forall x)(\forall y)(Rxy \supset {\sim}Ryx) \supset (\forall x){\sim}Rxx$	1-6 C.P.
8. $(\forall R)[(\forall x)(\forall y)(Rxy \supset {\sim}Ryx) \supset (\forall x){\sim}Rxx]$	7 UG

Task

Because most of the exercises at the end of Copi 5.5 (p. 155-156) have been used as examples in the text above, we ask you to attempt only two more by way a task:

I. Symbolise the following propositions:

10. Everyone has some unusual attribute or other. It would be an unusual person who had no unusual attributes. (Px: x is a person. Ux: is an unusual individual. $\boldsymbol{U}F$: F is an unusual attribute.)

II. Prove the following:

7. All intransitive binary relations are irreflexive.

Feedback

These solutions were again provided by John Ross. All of them for Copi 5.2-5 are available here.

10. $(\forall x)[Px \supset (\exists F)(\boldsymbol{U}F \bullet Fx)]$ and

$(\forall y)\{[Py \bullet (\forall G)(\boldsymbol{U}G \supset {\sim}Gy)] \supset Uy\}$ or $(\forall y)\{[Py \bullet {\sim}(\exists G)(\boldsymbol{U}G \bullet Gy)] \supset Uy\}$

7.

→ 1.	$(\forall x)(\forall y)(\forall z)[(Rxy \bullet Ryz) \supset {\sim}Rxz]$	(definition of intransitivity)
2.	$(\forall x)(\forall y)[(Rxy \bullet Ryx) \supset {\sim}Rxx]$	1 UI
3.	$(\forall x)[(Rxx \bullet Rxx) \supset {\sim}Rxx]$	2 UI
4.	$(Rxx \bullet Rxx) \supset {\sim}Rxx$	3 UI
5.	$Rxx \supset {\sim}Rxx$	4 Taut.
6.	${\sim}Rxx \vee {\sim}Rxx$	5 Impl.
7.	${\sim}Rxx$	6 Taut.
8.	$(\forall x){\sim}Rxx$	7 UG
9.	$(\forall x)(\forall y)(\forall z)[(Rxy \bullet Ryz) \supset {\sim}Rxz] \supset (\forall x){\sim}Rxx$	1-8 C.P.
10.	$(\forall R)\{(\forall x)(\forall y)(\forall z)[(Rxy \bullet Ryz) \supset {\sim}Rxz] \supset (\forall x){\sim}Rxx\}$	9. UG

Reference

COPI, I.M. (1979) *Symbolic Logic,* 5th Edition. Macmillan: New York

Acknowledgement: Unless otherwise acknowledged in the text, the bulk of explanations and examples above have been compiled from lecture notes on Symbolic Logic presented by the University of Cape Town by Professor Ian Bunting. The section on definite descriptions and the meaning of “the” were reconstructed from notes on the Philosophy of Language by Professor David Brooks at the same institution.

The next Critical Reasoning study unit concerns hypothesis testing.