Critical Reasoning 02 – Validity

Arguments which reason from one or more general statements to a specific conclusion are known as **deductive**. They are the darlings of Mathematicians and Logicians because if they are correctly structured then they absolutely guarantee the truth of the conclusion(s) if the premises are true. Deductive arguments that are correctly structured are said to be **valid**, such as the one about Socrates' mortality, recall:

1. All humans are mortal.
2. Socrates is human.
3 ∴ Socrates is mortal.

The first premise, as arranged here, that contains the universal statement is designated the **major premise** while the second one that contains the specific instance is referred to as the **minor premise**. Historically all such two premise and one conclusion arguments have been known as **syllogisms**. It is very easy to see why this particular argument is valid by imagining the set of all humans *H*. If all members of *H* are mortal and Socrates belongs to *H*, then he must also be mortal because he belongs to that set, all of whose members are mortal. What about the following argument?

1. All mammals can fly.
2. Pigs are mammals.
3. ∴ Pigs can fly.

Actually there is nothing wrong with the structure of this argument, in fact is has the same logical structure as the Socrates argument above, namely universal instantiation, therefore it is also valid. What *is* wrong with it is that the first premise is false, (only bats among all mammals can fly.) Arguments like the first example that are valid *and* have true premises are known as **sound** arguments, while those, like the second, whose premises are false, are known as **unsound**. Fortunately an unsound argument can always be rescued by doing a little research or fact checking to ensure that we get the premises to be true.

The following is an example of an argument that has true premises but a faulty structure that renders it not valid or **invalid**.

1. Cheetahs are fast.
2. Some cars are fast
3. ∴ Therefore some cars are cheetahs.

Invalid arguments such as this one are known as **fallacies** and no amount of fact checking can fix them, since they are structurally defective. We need to be vigilant that we aren't taken in by them or deploy them ourselves. For this reason Critical Reasoning 04 is devoted entirely to fallacies. For now though let us try and identify what exactly is wrong with the argument above. Just because two things share a common property (*e.g.* being fast) that does not necessarily mean that they are the same kind of thing, and that it what the above argument incorrectly assumes.

One **test for invalidity** in general runs as follows: Ask yourself if it is at all possible that all the premises of an argument could be true and yet its conclusion simultaneously false. If the answer is yes, then the argument in question is invalid, meaning that there is some defect in its structure. The argument above clearly passes the test for invalidity (or to put it another way, flunks the test for validity.) Here is why: while we know both premises to be true, we also know the conclusion to be false so it *is* possible for this argument to have both true premises and a simultaneously false conclusion.

While there are formal proofs of validity and invalidity they are all expressed in the language of Symbolic Logic, which we are yet to acquire, however this little test is sufficient to separate valid from invalid arguments in a way that is fool proof because it will never flag a valid argument as invalid or vice versa. *E.g.* Try to identify which of the following arguments are valid or invalid and also which are also sound. In the case that you do detect an invalid argument see if you can spot the structural defect.

a)
1. All animals live on Mars.
2. All humans are animals.
3. ∴ Therefore, all humans live on Mars.

b)
1. Every president is famous.
2. Gaga is famous.
3. ∴ Gaga is a president.

c)
1. Cherries are red.
2. Fire trucks are not cherries.
3. ∴ Fire trucks are not red.

d)
1. If someone hacked the system then he must have used a computer.
2. Mike doesn't even know how to use a computer.
3. ∴ Mike cannot have hacked the system.

e) 1. Either we are we are always certain or we shall have to question all our beliefs.
<u>2. We are most assuredly not always certain.</u>
3. ∴ We shall have to question all our beliefs.

f) 1. Cuddles is a kitten.
<u>2. All kittens are cats.</u>
3. ∴ Cuddles is a cat.

g) 1. If it shines we shall go for a walk and if it pours we shall stay indoors.
<u>2. Either it shines or it pours.</u>
3. ∴ Either we shall go for a walk or stay indoors.

Argument a) is a valid argument and therefore has no structural defect because, if indeed all animals *did* live on Mars and humans as we know are animals, then there is no way that humans could not simultaneously not live on Mars too. (Recall our test for invalidity – Make the premises true and try to spot if it's possible for the conclusion to be simultaneously false.) What is wrong with a) is not a matter of structure, but a matter of fact. The first premise is false therefore the argument is valid but unsound.

b) is an invalid argument. You don't even have to imagine a situation in which the premises are true and the conclusion is simultaneously false because in reality it is already so. To make clearer what is structurally wrong with this argument, let us rewrite the first premise as a **conditional**, (an "if… then…" statement,) thus:

1. If someone is a president then that someone is famous.
<u>2. Gaga is famous.</u>
3. ∴ Gaga is a president.

In general, with conditional statements, what comes after the "if" but before the "then" is known as the **antecedent**, while what goes after the "then" is known as the **consequent**. Here specifically, the problem is that second premise wants the first premise in reverse, (*i.e.* all famous people are presidents,) which we know to be false. This kind of invalid argument, or fallacy, is officially known as **affirming the consequent** because it tries to get at the antecedent of a conditional statement by affirming the consequent, to which we are not entitled.

c) is also an invalid argument. We know that both the premises are true, while the conclusion is simultaneously false, therefore the argument passes our test for invalidity (or flunks the test for validity.) Again for clarity, we can rewrite the first premise in conditional form, thus:

1. If it is a cherry then it is red
<u>2. Fire trucks are not cherries.</u>
3. ∴ Fire trucks are not red.

What is structurally wrong with this argument is similar to the one above, only in this case it tries to get the consequent by contradicting antecedent. This kind of fallacy, therefore is officially known as **denying the antecedent**.

d) is valid argument. You could not imagine a scenario where both the premises and the conclusion are not true *i.e.* where a system gets hacked via a computer and yet the guilty party is truly

computer illiterate. So this argument fails the invalidity (or passes the validity) test. Unfortunately we don't know who Mike is, so we can't say if the second premise is true, therefore we cannot without further information say whether this argument is sound or unsound.

e) is also a valid argument. It is not possible to imagine both the premises as true while the conclusion is simultaneously false, so again this argument fails the invalidity (or passes the validity) test. According to arguments of this sort, the first premise asserts that one or both of two possibilities obtain, while the second premise claims that one possibility does not obtain, so that in conclusion we may infer that the other one must.

f) is also a valid argument because it is not possible for the premises both to be true and yet for the conclusion to be simultaneously false. Using sets to represent the relations between the premises might be helpful if the validity of the argument is not immediately apparent. Consider the set of all kittens of which Cuddles is a member. If the set of all Cats including adult domestic Cats and all other felines also includes kittens, so that if a creature is a member of the set of all kittens it is also a member of the set of all Cats.

g) is also a valid argument because it is not possible for both of the premises to be true and the conclusion to be simultaneously false. The structure of this type of argument is a combination two simpler forms of argument such that if two conditionals are true, and one of their antecedents is true, then one of their consequents must also be true. Think of it this way, if there were only two conditional possibilities *i.e. if* shines *then* we shall go for a walk (and) *if* it pours *then* we shall stay indoors, then either shining or pouring will necessitate one of the consequences to be true *i.e.* going for a walk *or* staying indoors.

Elementary Rules of Inference: Of all the valid forms of arguments encountered so far each is ether a representative or a combination of two or more of the elementary valid arguments below (adapted from Copi, 1979.) Their validity will be proved in Critical Reasoning 07. Meanwhile we can use capital letters to stand for sentences just as lower case letters are used to stand for variables in Mathematics. For this reason Logic of this variety is often referred to as the Sentential Calculus (or the calculus of sentences.) Note that each elementary valid argument has a traditional name as well as a standard abbreviation which can be used in proofs.

1. *Modus ponens* (M.P.)
If A then B
A
∴ B

2. *Modus Tollens* (M.T.)
If A then B
not B
∴ not A

3. Hypothetical Syllogism (H.S.)
If A then B
If B then C
∴ If A then C

4. Disjunctive Syllogism (D.S)
A or B
not A
∴ B

5. Constructive Dilemma (C.D.)
If A then B and If C then D
A or C
∴ B or D

6. Destructive Dilemma (D.D.)
If A then B and If C then D
∴ not B or not D

7. Simplification (Simp.)
A and B
∴ A

8. Conjunction (Conj.)
A
B
∴ A and B

9. Addition (Add.)
A
∴ A or B

E.g. Try to name the elementary rule of valid inference at work in the following two arguments.

h) 1. If the bough breaks the baby will cry.
2. The baby does cry.
3. ∴ The bough broke.

i) 1. If I don't have transport I can't get to work.
2. If I can't get to work, I won't get paid.
3. ∴ If I don't have transport I won't get paid.

Argument h) looks a bit like *modus ponens* but actually it is not because it is invalid because you could imagine an instance in which the baby cries in response to something other than the bough breaking. Once again this is a case of the fallacy affirming the consequent. If we let "the bough breaks" = B and "the baby will cry" = C then this argument can be represented as:

If B then C
C
∴ B

which does not match any of elementary rule of valid inference for the simple reason that it is not valid.

Argument i) matches the form of a hypothetical syllogism, although is this case each of its sentences is negated, never the less we can let "I don't have transport" = not T and "I can't get to work" = not W and ''I won't get paid" = not P and represent the argument as follows:

If not T then not W
If not W then not P
∴ not P

which is a valid argument matching the form of a hypothetical syllogism.

Task

Now return to arguments a) to g) above and do the same exercise, *i.e.* to name the elementary rule of valid inference at work in each. Remember that fallacies will not match any of the nine named forms because they are invalid. If by now, you are wondering why we should be concerning ourselves with these named formal arguments at all, it is because 1) they are models of validity and 2) so that we shall not have to reinvent for ourselves, every time, what has already been proven.

Feedback

a) *Modus ponens*
b) Invalid - Affirming the consequent
c) Invalid - Denying the antecedent
d) Modus tollens
e) Disjunctive syllogism
f) Hypothetical syllogism
g) Constructive dilemma

In the following Critical Reasoning unit we shall be examining the nature of cogency and analogy.